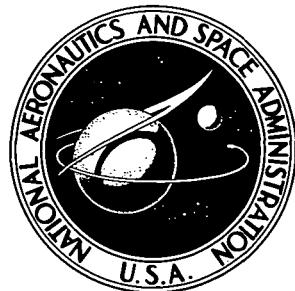


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NEW GENERALIZED RHEOLOGICAL MODEL  
FOR LUBRICATION OF A BALL  
SPINNING IN A NONCONFORMING GROOVE

by Charles W. Allen, Dennis P. Townsend,  
and Erwin V. Zaretsky

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16. Abstract <p>The previously developed elastohydrodynamic theory for predicting the spinning friction of a ball in a nonconforming groove was modified to incorporate a new rheological model. The new rheological model is based on the exponential pressure viscosity relation for low shear stresses, but at high shear rates and pressures the relation is altered to one in which the shear stress is proportional to the normal stress. The model was fitted to experimental spinning torques for four different lubricants: a synthetic paraffinic lubricant, di-2-ethylhexyl sebacate, a super-refined naphthenic mineral oil, and a polyphenyl ether (5P4E). Good agreement between the model and experiment was found.</p>			
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# NEW GENERALIZED RHEOLOGICAL MODEL FOR LUBRICATION OF A BALL SPINNING IN A NONCONFORMING GROOVE

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## SUMMARY

A new rheological model incorporating four parameters: ambient viscosity, pressure viscosity coefficient, a lubricant factor, and transition shear stress was developed to describe the traction for a spinning point contact under elastohydrodynamic conditions. The model is based on an exponential pressure viscosity model for shear stresses lower than  $6.89 \times 10^6$  newtons per square meter (1000 psi). Above this shear stress, the exponential law is assumed to be followed until the ratio of shear/normal stress exceeds a certain value called the lubricant factor which is a pseudo-coefficient of friction. For all higher pressures, the shear stress is obtained by taking the product of the lubricant factor and normal stress.

The analysis was compared with experimental spinning torque data reported in reference 7 for a synthetic paraffinic oil, a di-2-ethylhexyl sebacate, a super-refined naphthenic mineral oil, and a polyphenyl ether lubricant. Good correlation between theoretical and experimental results was obtained for these lubricants. The rheological model which limits the shear at high pressures but allows for low shear stresses in the micro-slip region, enables the spinning torque to be predicted to a sufficient degree of accuracy for most applications.

## INTRODUCTION

In recent years, a complete evolution has occurred in ball bearing design and analysis. Early work assumed that at the ball-race contact, spinning and rolling would occur at one raceway and only pure rolling at the other raceway (refs. 1 and 2). This work was based on the premise that there was a constant coefficient of friction at the ball-race contacts. Work reported in reference 3 and subsequently in references 4 and 5 indicated that the friction was not dependent upon a single value of friction coefficient but varied

according to the magnitude of the contact stress, contact geometry, and lubricant type and viscosity. As a result, no simple analysis utilizing a constant value for the coefficient of friction could be applied to a ball-race contact. Reference 6 recognizes this phenomenon and presents an analysis for the kinematics of a ball bearing considering elastohydrodynamic effects and assumed lubricant rheological properties. However, these properties are not completely defined, resulting in some second-order inaccuracies in the analysis. The authors of reference 7 developed a theoretical analysis based on the elastohydrodynamic theory of lubrication for spinning torque. This model provided close agreement with the experimental data reported (ref. 7).

In developing the analysis of reference 7 it was found that the assumption of a Newtonian fluid with an exponential pressure viscosity relation gave impossibly high values of torque. It was necessary therefore to introduce a cutoff point at which the pressure viscosity exponent decreased to a much lower value. The model of reference 7 yielded satisfactory results when the calculated shear rate was of the order of  $10^6$  reciprocal seconds. Similar models were applied to the lubricants investigated experimentally in reference 3 and the results are reported in reference 8.

Comparison with data of other researchers (ref. 9) showed the cutoff to occur at a much higher pressure when the film thickness was much greater than those of references 3 and 7. In other words, a lower shear rate resulted in a higher cutoff pressure.

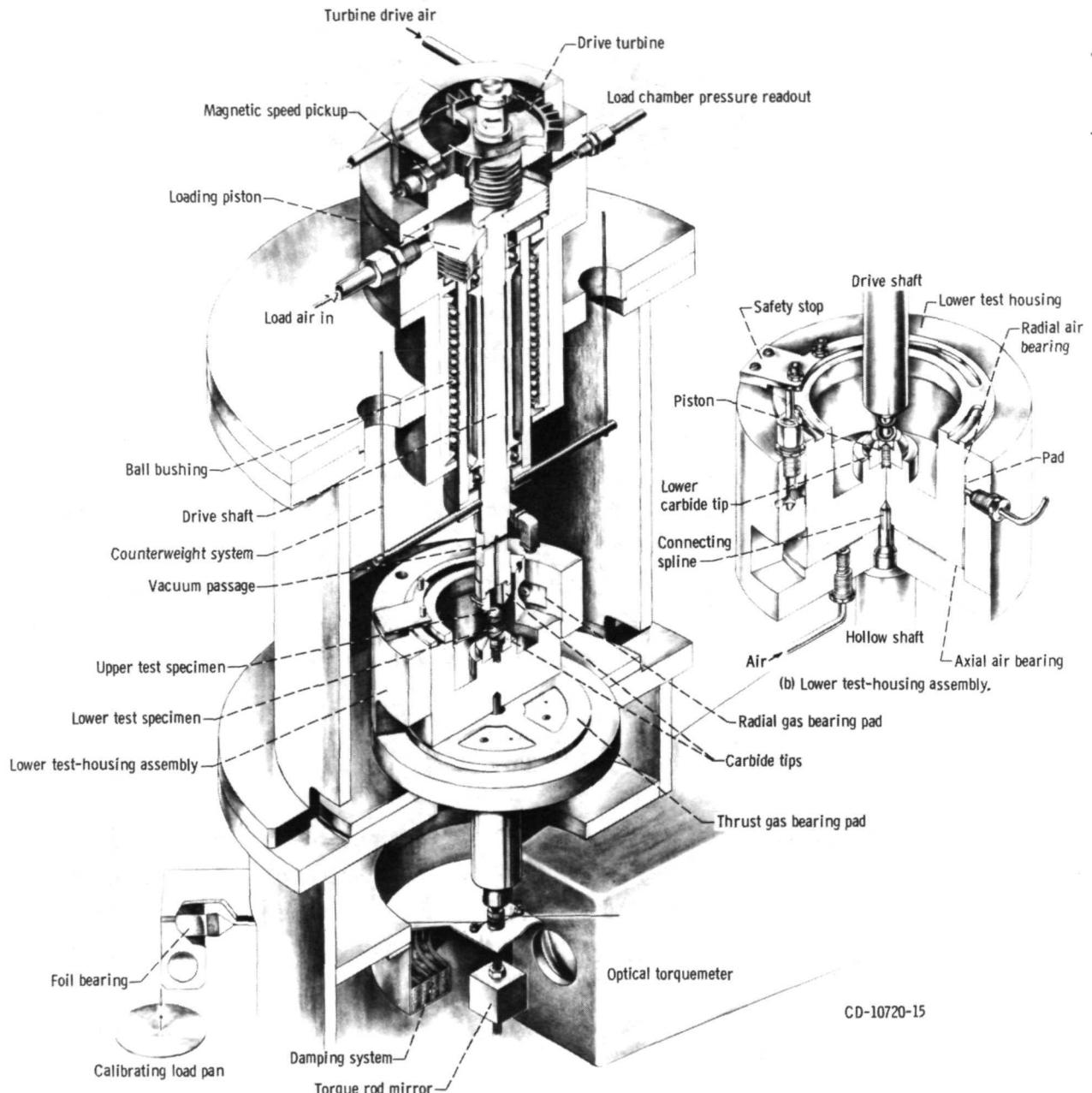
The objective of the work reported herein was to modify the composite viscosity model to take into account the shear rate dependency effects. The theory using the modified model will be compared with the experimental results for a synthetic paraffinic lubricant reported in reference 7 and with the experimental results for a di-2-ethylhexyl sebacate, a super-refined naphthenic mineral oil, and a polyphenyl ether as reported in reference 3.

The modified rheological model was suggested by remarks made by K. L. Johnson in a formal discussion presented in reference 7. He stated that "at high pressure, the film exhibits a critical shear stress which is approximately proportional to the pressure and decreases slightly with temperature, somewhat similar to a granular solid. This hypothesis leads to an approximately constant 'coefficient of friction'  $\tau/s$  independent of film thickness, as would be observed in dry sliding or boundary friction."

## APPARATUS AND SPECIMENS

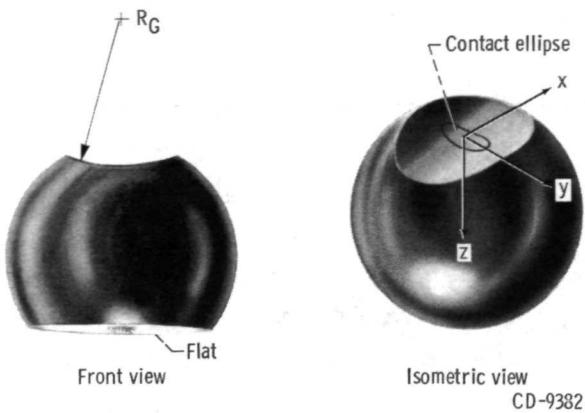
### Spinning Torque Apparatus

A spinning torque apparatus (see figs. 1(a) and (b)) described previously in references 3 and 4 was also used for the tests reported herein. The apparatus essentially consists of a turbine drive, a pneumatic load device, an upper and lower test specimen,



(a) General cutaway view.

Figure 1. - Spinning-torque apparatus.



(c) Lower test specimen.

Figure 1. - Concluded.

a lower test-housing assembly incorporating a hydrostatic airbearing, and a torque-measuring system. In operation, the upper test specimen is pneumatically loaded against the lower test specimen through the drive shaft. As the drive shaft is rotated, the upper test specimen spins in the groove of the lower test specimen. This causes an angular deflection of the lower test-specimen housing. This angular movement is sensed optically by the torque-measuring system and is converted into a torque value. During a test, the torque is continuously recorded on a strip chart.

## Specimens

The upper test specimen is a conventional 12.7-millimeter- (1/2-in.-) diameter bearing ball made of SAE 52100 steel. The ball has a nominal Rockwell C hardness of 61 and a surface finish of  $5 \times 10^{-8}$  meters ( $2 \mu\text{in.}$ ) rms. The lower test specimen (fig. 1(c)) is a 12.7-millimeter- (1/2-in.-) diameter ball from the same heat of material as the upper test specimen. The lower ball is modified by grinding a flat on one side and a cylindrical groove of radius  $R_G$  (fig. 1(c)) on the other. The groove simulates the race groove of a bearing. The axis of the groove is parallel to the flat. The groove radius expressed as a percentage of the upper-ball diameter is defined as the ball-race conformity. The specimens used in these tests were ground to ball-race conformities of 51, 55, and 60 percent. The surface finish of the cylindrical groove was approximately  $5 \times 10^{-8}$  to  $15 \times 10^{-8}$  meter ( $2$  to  $6 \mu\text{in.}$ ) rms.

## THEORETICAL ANALYSIS

An elastohydrodynamic analysis for the determination of the torque of a ball spinning in a nonconforming groove, shown in figure 1, was developed in reference 7. In the analysis of reference 7, the area of contact is elliptical, as shown in figure 2(a). In this ellipse an elastohydrodynamic film can theoretically be predicted, due to the spinning velocity, except within the inscribed circle of radius  $b$ . The frictional force over the ellipse is determined by making the system into a number of elemental rollers of width  $dy$  as shown in figure 2(b), and determining the frictional force on each roller. These rollers slide into and out of the contact region as the major axis of the ellipse is crossed.

The torque due to the region outside the inscribed circle and inside the contact ellipse is computed by integrating the moments of the elemental rollers over this region; that is,

$$M_1 = 2 \int_b^a y dF \quad (1)$$

To determine the frictional force on each elemental roller, it is split into elements of width  $dx$ . The friction force on each elemental roller is thus given by

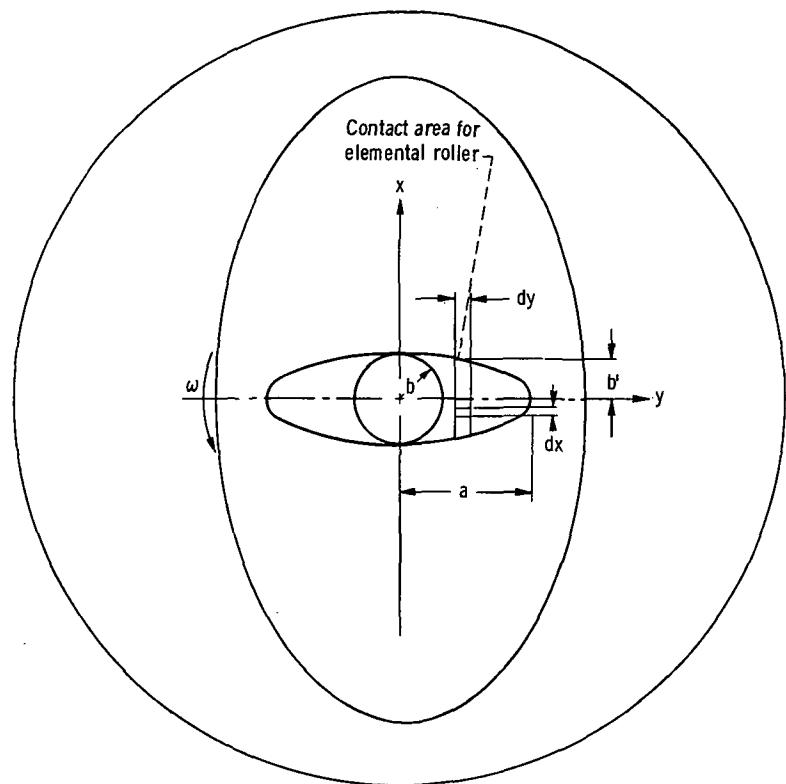
$$dF = 2 \left( \int_0^{b'} \tau dx \right) dy \quad (2)$$

where  $\tau$  is shear stress. For a Newtonian fluid, assuming a linear velocity gradient and ignoring the velocity in the  $x$  direction which is small, the approximate shear stress is given by

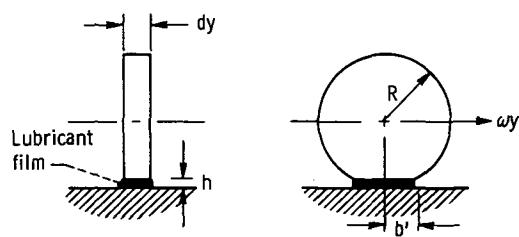
$$\tau = \mu \frac{\omega y}{h} \quad (3)$$

where  $h$ , the film thickness, is given by reference 10 for a roller and modified for each elemental roller by

$$h = \frac{1.6\alpha^{0.6} E^{0.03} R^{0.43}}{W^{0.13}} \left( \frac{\mu_0 \omega y}{2} \right)^{0.7} \quad (4)$$



(a) Contact ellipse for ball in nonconforming groove.



(b) Elemental roller.

Figure 2. - Ground lower ball showing contact ellipse and elemental roller.

where

$$E' = \frac{E}{1 - \sigma^2} \quad (5)$$

For steel,  $E'$  is  $22.3 \times 10^{10}$  newtons per square meter ( $32.3 \times 10^6$  psi).

The assumption of a Hertzian stress distribution leads to a load distribution  $W$ ; that is,

$$W = \frac{0.75}{a} P \left[ 1 - \left( \frac{y}{a} \right)^2 \right] \quad (6)$$

where  $P$  is the normal load.

Using the exponential pressure viscosity relation, the viscosity at any location  $x, y$  would be given by

$$\mu = \mu_0 e^{\alpha S} \quad (7)$$

where  $S$  is the contact pressure at that point. If a Hertzian distribution is assumed, the contact pressure is given by

$$S = \frac{1.5P}{\pi ab} \left( 1 - \frac{x^2}{b^2} - \frac{y^2}{a^2} \right)^{1/2} \quad (8)$$

The proposed rheological model is based on the assumption that the foregoing relations are valid at low shear stresses but a point is reached above which the shear stress becomes proportional to the normal stress. These conditions may be represented by the following relations:

$$\tau = \mu_0 e^{\alpha S} \frac{\omega y}{h} \quad (9a)$$

$$\tau = fS \quad \text{and} \quad \mu_0 e^{\alpha S} \frac{\omega y}{h} > fS \quad (9b)$$

The introduction of a lubricant factor  $f$  serves to limit the shear stress at high pressures and shear rates to a fraction of the normal stress. However, there are cases at low pressures where the shear stress is greater than  $fS$ . Where this occurs, it is more reasonable to use the viscous model. Therefore, a transitional shear stress  $\tau_c$  can be introduced to allow for large values of shear stress at low pressure where the viscous model shows a shear stress greater than  $fS$ . Thus, an additional condition is imposed wherein equation (9a) should be used where

$$\tau < \tau_c \quad \text{and} \quad \tau_c < \tau < fS$$

In order to visualize the behavior of the fluid under the foregoing conditions, figure 3 shows a plot of shear stress as a function of normal stress (pressure) for a synthetic paraffinic lubricant at various shear rates. For purposes of comparison, the shear stresses computed by equations (9a) and (9b) may be substituted into equation (3), and an

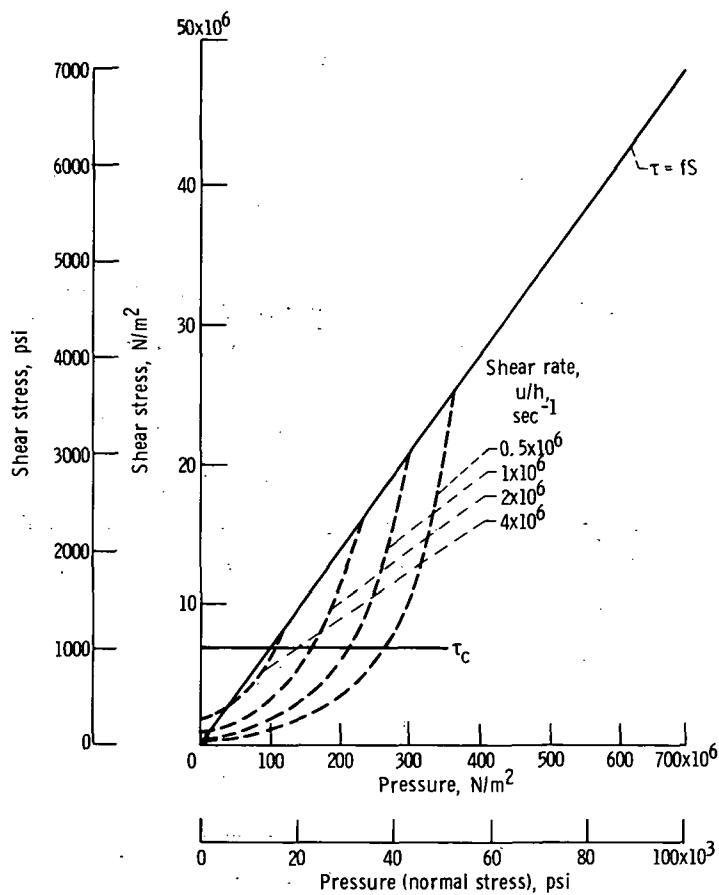


Figure 3. - Shear stress plotted against pressure for synthetic paraffinic lubricant at various shear rates.

equivalent viscosity may be obtained for any shear rate. Under the method of reference 7 this form of the equivalent viscosity would be used to calculate the spinning torque. Figure 4 shows this equivalent viscosity for various shear rates and also a comparison with the previous rheological model from reference 7. The previous model is seen to be approximately equivalent to a special case of the newer, more general, model.

Computation of the moment due to the outer region of the ellipse is undertaken by using equations (1) and (2). Equation (4) is used to compute the film thickness. The rheological relations (eqs. (9a) and 9(b)) are used to compute the shear stress at each point. By combining and integrating these equations twice numerically, the moment about the z axis is obtained.

There remains the area within the inscribed circle of radius b where a conventional elastohydrodynamic film theoretically is impossible to maintain. However, separation of

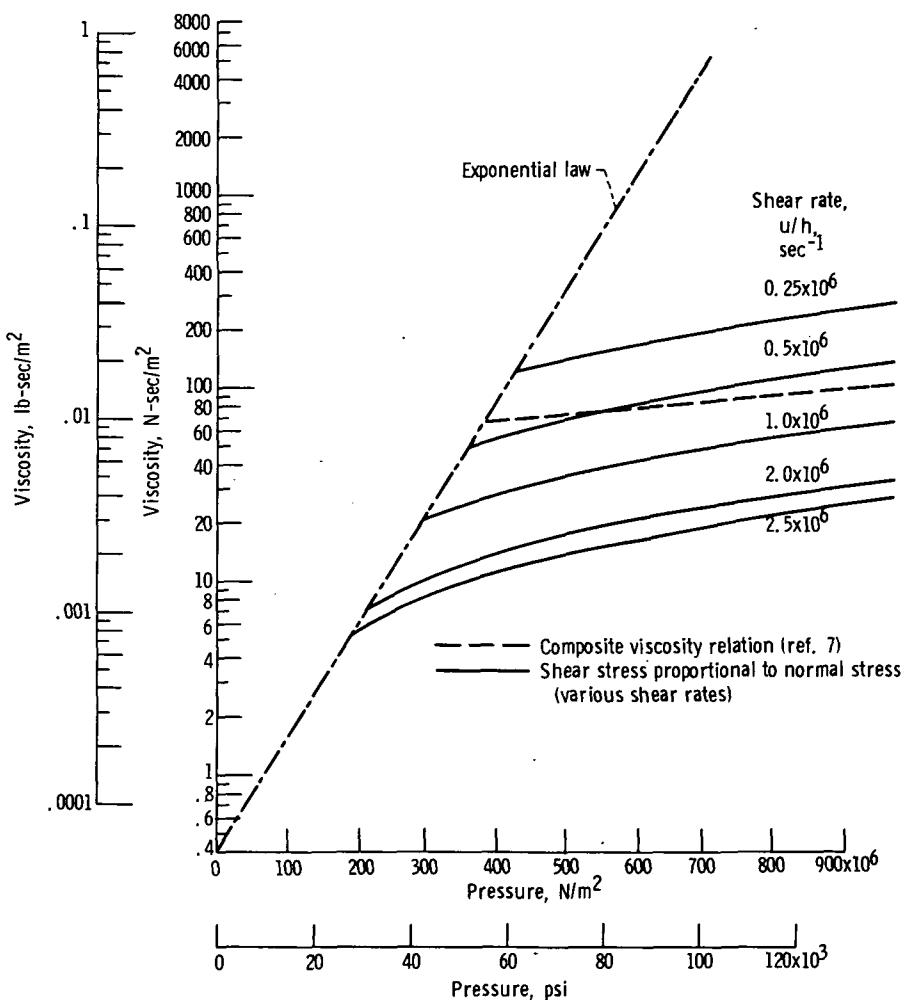


Figure 4. - Equivalent viscosity plotted against pressure using new rheological model for synthetic paraffinic lubricant at various shear rates.

the surfaces is made possible by means of a microasperity elastohydrodynamic film (ref. 11) or squeeze film. Because the area outside the inscribed circle from  $y = -b$  to  $+b$  is small, it is neglected. If the inscribed circle is then divided into elemental rings of radius  $r$  and width  $dr$ , the torque  $M_2$  for the inscribed circle becomes

$$M_2 = 2\pi \int_0^b \tau_b r^2 dr \quad (10)$$

The moment over the inscribed circle is then

$$M_2 = \frac{2\pi \tau_b b^3}{3} \quad (11)$$

In addition to the effect of the lubricant within the Hertzian contact region, the effect of the viscous drag of the lubricant outside this region must also be considered (fig. 5). The expression for this moment is given in reference 5 as

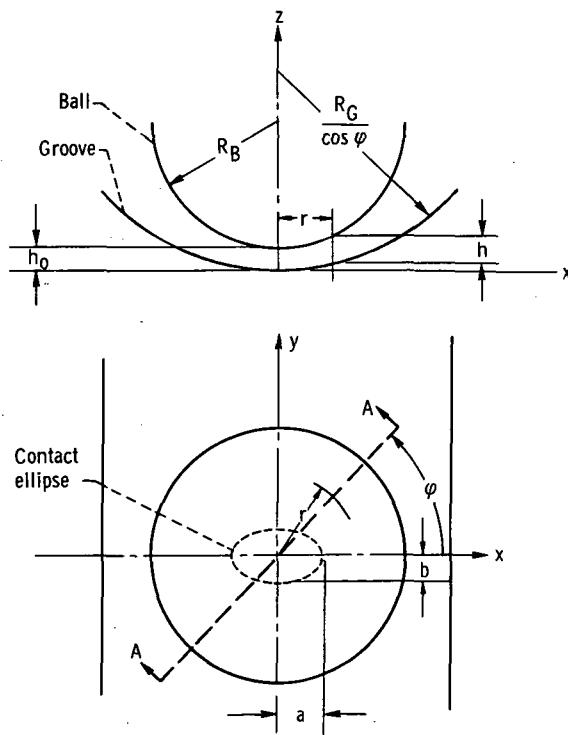


Figure 5. - Schematic of ball-race contact.

$$M_3 = 4\mu\omega \int_0^{\pi/2} \int_0^{KR_B} \frac{r^3 d\varphi dr}{\left(R_B + h_o - \frac{R_G}{\cos \varphi}\right) + \left[\left(\frac{R_G}{\cos \varphi}\right)^2 - r^2\right]^{1/2} - \left(R_B^2 - r^2\right)^{1/2}} \quad (12)$$

This may be integrated numerically over the region outside the contact ellipse. The total spinning torque  $M_s$  is therefore the sum of the moments given by equations (1), (11), and (12)

$$M_s = M_1 + M_2 + M_3 \quad (13)$$

## RESULTS AND DISCUSSION

The spinning moments for a ball spinning in a nonconforming groove with a synthetic paraffinic lubricant and conformities of 51, 55, and 60 percent were computed using the revised rheological model and compared with the experimental values from reference 7. The theoretical results which are given in figure 6 show good agreement with the experimental data of reference 7 which are shown for comparative purposes. The lubricant parameters used in the rheological model were as follows:

Ambient viscosity, $\mu_o$ , N-sec/m <sup>2</sup> (lb-sec/in. <sup>2</sup> ) . . . . .	0.414 ( $6 \times 10^{-5}$ )
Pressure viscosity coefficient, $\alpha$ , m <sup>2</sup> /N (psi <sup>-1</sup> ) . . . . .	$1.33 \times 10^{-8}$ ( $0.92 \times 10^{-4}$ )
Transitional shear stress, $\tau_c$ , N/m <sup>2</sup> (psi) . . . . .	$6.89 \times 10^6$ (1000)
Lubricant factor, f . . . . .	0.07

(The preceding values for viscosity and pressure viscosity coefficient were the same as reported in ref. 7.)

For the di-2-ethylhexyl sebacate lubricant tests which were reported in reference 3, the following fluid parameters were used:

Ambient viscosity, $\mu_o$ , N-sec/m <sup>2</sup> (lb-sec/in. <sup>2</sup> ) . . . . .	0.016 ( $0.24 \times 10^{-5}$ )
Pressure viscosity coefficient, $\alpha$ , m <sup>2</sup> /N (psi <sup>-1</sup> ) . . . . .	$1.45 \times 10^{-8}$ ( $1.0 \times 10^{-4}$ )
Transitional shear stress, $\tau_c$ , N/m <sup>2</sup> (psi) . . . . .	$6.89 \times 10^6$ (1000)
Lubricant factor, f . . . . .	0.045

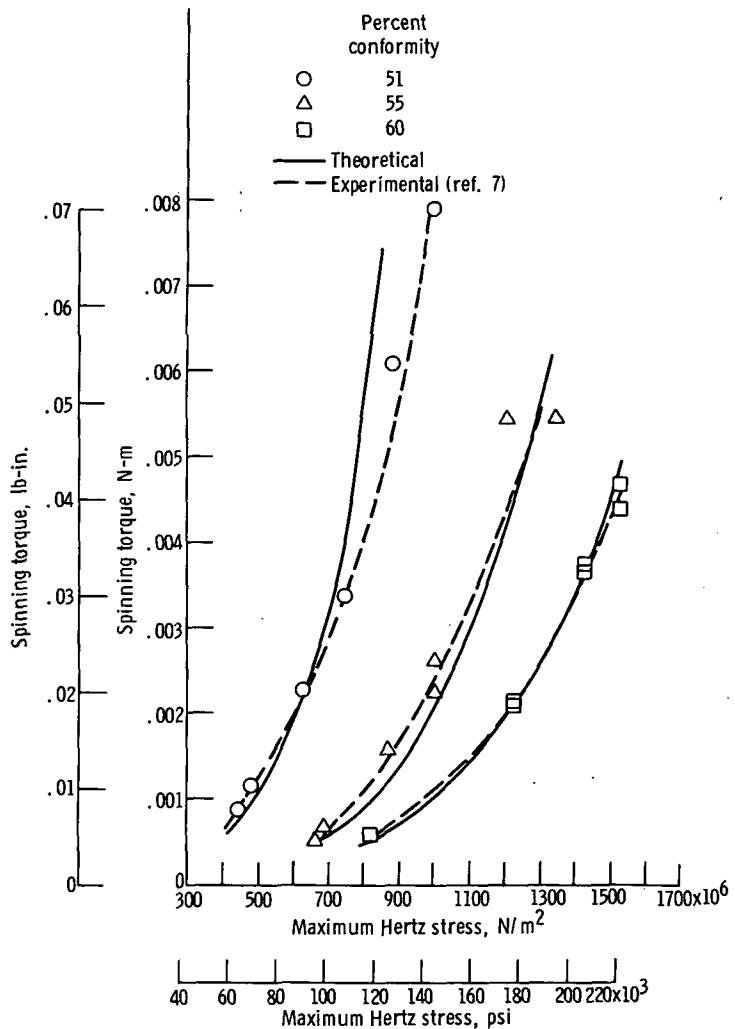


Figure 6. - Comparison of experimental and theoretical spinning torque for synthetic paraffinic lubricant for conformities of 51, 55, and 60 percent. Ambient viscosity  $\mu_0 = 0.414 \text{ N}\cdot\text{sec}/\text{m}^2 (6 \times 10^{-5} \text{ lb}\cdot\text{sec}/\text{in}^2)$ ; pressure viscosity coefficient  $\alpha = 1.33 \times 10^{-8} \text{ m}^2/\text{N}$  ( $0.92 \times 10^{-4} \text{ psi}^{-1}$ ); transitional shear stress  $\tau_c = 6.89 \times 10^6 \text{ N}/\text{m}^2 (1000 \text{ psi})$ ; lubricant factor  $f = 0.07$ ; spinning speed, 1050 rpm.

The value of  $\alpha$  for the di-2-ethylhexyl sebacate is that reported in reference 12. The experimental and theoretical curves which are shown in figure 7 are in good agreement.

For the super-refined naphthenic mineral oil, good agreement was obtained between the experimental results from reference 3 and theoretical results using the rheological model with the following parameters:

Ambient viscosity, $\mu_0$ , N-sec/m <sup>2</sup> (lb-sec/in. <sup>2</sup> ) . . . . .	0.079 ( $1.14 \times 10^{-5}$ )
Pressure viscosity coefficient, $\alpha$ , m <sup>2</sup> /N (psi <sup>-1</sup> ) . . . . .	$2.15 \times 10^{-8}$ ( $1.48 \times 10^{-4}$ )
Transitional shear stress, $\tau_c$ , N/m <sup>2</sup> (psi) . . . . .	$6.89 \times 10^6$ (1000)
Lubricant factor, $f$ . . . . .	0.075

The value of  $\alpha$  used here is the same as reported in reference 13. These results are shown in figure 8.

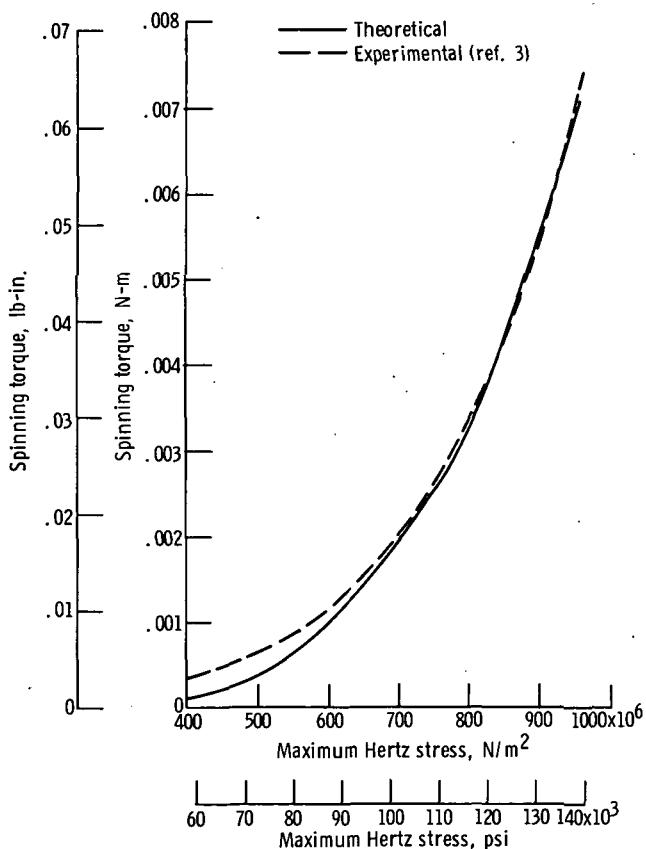


Figure 7. - Comparison of experimental and theoretical spinning torque for di-2-ethylhexyl sebacate. Ambient viscosity  $\mu_0 = 0.016$  N-sec/m<sup>2</sup> ( $0.24 \times 10^{-5}$  lb-sec/in.<sup>2</sup>); pressure viscosity coefficient  $\alpha = 1.45 \times 10^{-8}$  m<sup>2</sup>/N ( $1.0 \times 10^{-4}$  psi<sup>-1</sup>); transitional shear stress  $\tau_c = 6.89 \times 10^6$  N/m<sup>2</sup> (1000 psi); lubricant factor  $f = 0.045$ ; spinning speed, 950 rpm; conformity, 51 percent.

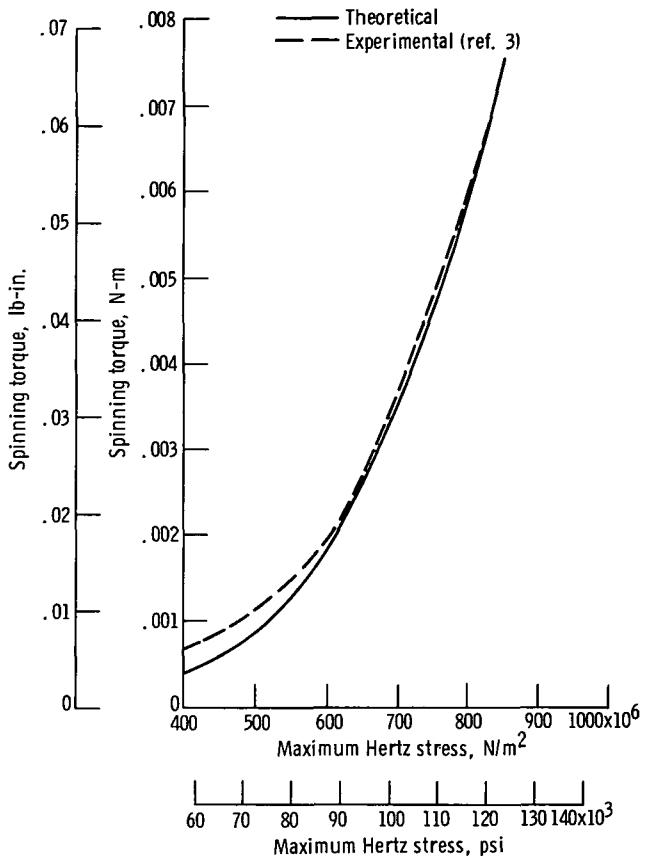


Figure 8. - Comparison of experimental and theoretical spinning torques for super-refined naphthenic mineral oil. Ambient viscosity  $\mu_0 = 0.079 \text{ N-sec/m}^2 (1.14 \times 10^{-5} \text{ lb-sec/in.}^2)$ ; pressure viscosity coefficient  $\alpha = 2.15 \times 10^{-8} \text{ m}^2/\text{N}$  ( $1.48 \times 10^{-4} \text{ psi}^{-1}$ ); transitional shear stress  $\tau_c = 6.89 \times 10^6 \text{ N/m}^2$  (1000 psi); lubricant factor  $f = 0.075$ ; spinning speed, 950 rpm; conformity, 51 percent.

The comparison between the experimental (ref. 3) and theoretical results for the polyphenyl ether is shown in figure 9. While there was reasonably good correlation between the theoretical and the experimental results, it was not possible to obtain as good a correlation as with the other fluids even though a number of different lubricant parameters were tried. The parameters giving the best correlations are as follows:

Ambient viscosity, $\mu_0$ , $\text{N-sec/m}^2$ ( $\text{lb-sec/in.}^2$ ) . . . . .	0.8 ( $11.6 \times 10^{-5}$ )
Pressure viscosity coefficient, $\alpha$ , $\text{m}^2/\text{N}$ ( $\text{psi}^{-1}$ ) . . . . .	$4.64 \times 10^{-8}$ ( $3.2 \times 10^{-4}$ )
Transitional shear stress, $\tau_c$ , $\text{N/m}^2$ ( $\text{psi}$ ) . . . . .	$6.89 \times 10^6$ (1000)
Lubricant factor, $f$ . . . . .	0.07

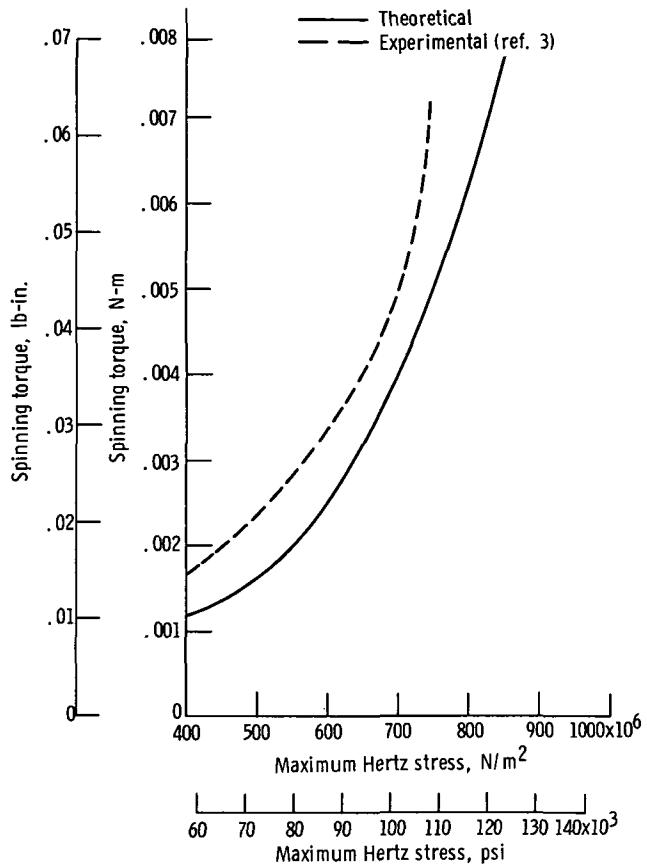


Figure 9. - Comparison of theoretical and experimental spinning torques for polyphenyl ether (5P4E). Ambient viscosity  $\mu_0 = 0.8 \text{ N-sec/m}^2$  ( $11.6 \times 10^{-5} \text{ lb-sec/in.}^2$ ); pressure viscosity coefficient  $\alpha = 4.64 \times 10^{-8} \text{ m}^2/\text{N}$  ( $3.2 \times 10^{-4} \text{ psi}^{-1}$ ); transitional shear stress  $\tau_c = 6.89 \times 10^6 \text{ N/m}^2$  (1000 psi); lubricant factor  $f = 0.07$ ; spinning speed, 950 rpm; conformity, 51 percent.

(The preceding value of  $\alpha$  is that given in ref. 14.)

## CONCLUDING REMARKS

The results of this study have shown that the traction in a sliding elastohydrodynamic contact may be computed on the basis of four fluid parameters. Of these parameters only the following three are critical:

- (1) Ambient viscosity  $\mu_0$
- (2) Pressure viscosity coefficient  $\alpha$
- (3) Lubricant factor  $f$

The fourth parameter, the transitional shear stress  $\tau_c$ , is merely a device to allow the

Newtonian viscosity relation to be applied at low pressure when the ratio  $\tau/S$  is greater than  $f$  and may also be greater than unity. An identical value for the transition shear stress  $\tau_c$  ( $6.89 \times 10^6$  N/m<sup>2</sup> (1000 psi)) was used for all lubricants in this study.

The pressure viscosity coefficient used in the analysis was, in each case, taken from published values. The lubricant factor was adjusted until good correlation was obtained. In general, a lubricant factor in the range 0.07 to 0.075 gave good correlation. The exception to this was the di-2-ethylhexyl sebacate for which a lubricant factor of 0.045 was found. The rheological model enables the spinning torque to be predicted to a sufficient degree of accuracy for most applications.

## SUMMARY OF RESULTS

A new rheological model incorporating four parameters: ambient viscosity, pressure-viscosity coefficient, a lubricant factor and transition shear stress was developed to describe the traction for a spinning point contact under elastohydrodynamic conditions.

The new rheological model is based on an exponential pressure-viscosity model for shear stresses lower than 6.89 newtons per square meter (1000 psi). Above this shear stress, the exponential law is assumed to be followed until the ratio of shear/normal stress exceeds a certain value called the lubricant factor which is a pseudo-coefficient of friction. For all higher pressures, the shear stress is obtained by taking the product of the lubricant factor and normal stress.

The analysis was compared with experimental spinning torque data reported in references 3 and 7 for a synthetic paraffinic oil, a di-2-ethylhexyl sebacate, a super-refined naphthenic mineral oil, and a polyphenyl ether lubricant. The following results were obtained:

1. Good correlation between theoretical and experimental results was obtained for the synthetic paraffinic oil, the di-2-ethylhexyl sebacate, the super-refined naphthenic mineral oil, and the polyphenyl ether lubricant.
2. The rheological model which limits the shear at high pressures but allows for low shear stresses in the microslip region, enables the spinning torque to be predicted to a sufficient degree of accuracy for most applications.

Lewis Research Center,  
National Aeronautics and Space Administration,  
Cleveland, Ohio, February 20, 1973,  
501-24.

## APPENDIX - SYMBOLS

a	major semiaxis of contact ellipse, m (in.)
b	minor semiaxis of contact ellipse, m (in.)
b'	semiwidth of contact ellipse at $y$ , m (in.)
E	modulus of elasticity, $N/m^2$ (psi)
E'	materials properties factor, $N/m^2$ (psi)
F	friction force
f	lubricant factor
h	film thickness, m (in.)
$h_b$	value of $h$ at $x = b$ , m (in.)
$h_o$	minimum distance between ball and groove, m (in.)
K	constant defining outer boundary of integration
$M_s$	total spinning torque in Hertzian ellipse, N-m (lb-in.)
$M_1$	spinning torque in Hertzian ellipse outside inscribed circle, N-m (lb-in.)
$M_2$	spinning torque in Hertzian ellipse inside inscribed circle, N-m (lb-in.)
$M_3$	spinning torque due to viscous drag outside Hertzian ellipse, N-m (lb-in.)
P	normal load, N (lb)
R	radius of equivalent cylinder, m (in.)
$R_B$	radius of ball, m (in.)
$R_G$	radius of groove, m (in.)
r	polar coordinate
$r_o$	value of $r$ at outer boundary of Hertzian ellipse, m (in.)
S	contact stress, $N/m^2$ (psi)
u	relative velocity, m/sec (in./sec)
W	load per unit width, N/m (lb/in.)
x,y,z	Cartesian coordinates, m (in.)
$\alpha$	pressure viscosity exponent
$\mu$	absolute viscosity, $N\text{-sec}/m^2$ (lb-sec/in. <sup>2</sup> )
$\mu_o$	ambient viscosity, $N\text{-sec}/m^2$ (lb-sec/in. <sup>2</sup> )

$\sigma$  Poisson's ratio

$\tau$  shear stress,  $\text{N/m}^2$  (psi)

$\tau_b$  value of  $\tau$  at  $x = b$ ,  $\text{N/m}^2$  (psi)

$\tau_c$  transition shear stress,  $\text{N/m}^2$  (psi)

$\varphi$  polar coordinate

$\omega$  angular velocity, rad/sec

## REFERENCES

1. Jones, A. B.: Ball Motion and Sliding Friction in Ball Bearings. *J. Basic Eng.*, vol. 81, no. 1, Mar. 1959, pp. 1-12.
2. Jones, A. B.: A General Theory for Elastically Constrained Ball and Radial Roller Bearings Under Arbitrary Load and Speed Conditions. *J. Basic Eng.*, vol. 82, no. 2, June 1960, pp. 309-320.
3. Miller, Steven T.; Parker, Richard J.; and Zaretsky, Erwin V.: Apparatus for Studying Ball Spinning Friction. *NASA TN D-2796*, 1965.
4. Dietrich, Marshall W.; Parker, Richard J.; and Zaretsky, Erwin V.: Effect of Ball-Race Conformity on Spinning Friction. *NASA TN D-4669*, 1968.
5. Dietrich, M. W.; Parker, R. J.; Zaretsky, E. V.; and Anderson, W. J.: Contact Conformity Effects on Spinning Torque and Friction. *J. Lub. Tech.*, vol. 91, no. 2, Apr. 1969, pp. 308-313.
6. Harris, T. A.: An Analytical Method to Predict Skidding in Thrust-Loaded, Angular-Contact Ball Bearings. *J. Lub. Tech.*, vol. 93, no. 1, Jan. 1971, pp. 17-24.
7. Allen, C. W.; Townsend, D. P.; and Zaretsky, E. V.: Elastohydrodynamic Lubrication of a Spinning Ball in a Nonconforming Groove. *J. Lub. Tech.*, vol. 92, no. 1, Jan. 1970, pp. 89-96.
8. Allen, Charles W.; Townsend, Dennis P.; and Zaretsky, Erwin V.: Comparison of Conventional and Microasperity Elastohydrodynamic Lubrication of a Ball Spinning in a Nonconforming Groove. *NASA TN D-6761*, 1972.
9. Johnson, K. L.; and Cameron, R.: Shear Behaviour of Elastohydrodynamic Oil Films at High Rolling Contact Pressures. *Proc. Inst. Mech. Eng.*, vol. 182, no. 14, 1967-68, pp. 307-330.
10. Dowson, D.; and Higginson, G. R.: Elastohydrodynamic Lubrication. Pergamon Press, 1966, p. 96.
11. Allen, Charles W.; and Zaretsky, E. V.: Microasperity Model for Elastohydrodynamic Lubrication of a Spinning Ball on a Flat Surface. *NASA TN D-6009*, 1970.
12. Research Committee on Lubrication: Pressure-Viscosity Reports. Vols. I and II. ASME, 1953.
13. Foord, C. A.; Hammann, W. C.; and Cameron, A.: Evaluation of Lubricants Using Optical Elastohydrodynamics. *ASLE Trans.*, vol. 11, no. 1, Jan. 1968, pp. 31-43.

14. Klaus, E. E.; Tewksbury, E. J.; and Fenske, M. R.: Fluids, Lubricants, Fuels and Related Materials. Pennsylvania State Univ., (AFML TR 65-112, AD-467077), Apr. 1965.

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